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THE NATURE OF PRIMARY QUALITIES.

WHAT is the 'real size' of an object, as geometry and mechanics consider it? Is it its visible or (as Berkeley called it) its 'tangible' extension? Berkeley adduces some excellent reasons why it cannot be the former, and concludes that it must be the latter. The visible extension is exceedingly variable, according to the position of the observer. The real extension of things no doubt varies, but not in any such fashion as that. The visible extension varies, too, according as we look at the thing through the naked eye or through the right or wrong end of a microscope.

The trouble is that very much the same objections can be made to Berkeley's theory, that the mathematical extension is tangible extension. For this varies greatly according to the part of the body employed. If Berkeley had noted such simple facts as the apparent separation of two fingers as they are drawn across the lips, or the enormous size of a cavity as the tongue-tip finds it in the tooth, he would not have gone astray here. It is as impossible in the case of tangible extension as in that of visible extension, to pick out any one of the various perceptions and regard it as characterizing the object.

Now in the case of a secondary quality (such as color) it can be urged, with much plausibility if not with convincing force, that science has no need for the conception of a 'real' quality. But in the case of extension this certainly cannot be said. Geometry and mechanics stand ready to refute such an assertion. But here we must beware of a misunderstanding. Geometry, as a branch of pure mathematics, has nothing in particular to do with the space of our experience. It can, indeed, in some of its forms, be applied to our space, but it can also be applied to many other things. Time, for example, makes a very good one-dimensional continuum, and it can easily be combined with three-dimensional space to make a four-dimensional continuum—as, indeed, is very commonly done (*e. g.*, in electro-magnetic theory). And the like may be said of pure mechanics. When, therefore, I say that the existence of geometry and mechanics compels the acceptance of the notion of a real extension of things, I mean not the pure sciences but the much older applied sciences, based upon inductively established principles. For space is not simply the field of a certain type of relations. Real things extend and move in space according to well-defined empirical uniformities. The hoop around a barrel is a little more than three times the distance across it. The baseball fielder 'judges' the batted ball correctly ninety-nine times out of a hundred.

The idea of experimentation in mechanics is familiar to us; indeed, the elements of the science are still commonly taught in this way. But it is so long since geometry reached the deductive stage, that the idea of geometrical experimentation seems fantastic. Logically, however, both sciences are on the same level in this respect. On the one hand, the first principles of both sciences are capable of being stated in a generalized form, which turns out, on analysis, to be nothing but a complicated resultant of the laws of formal logic. On the other hand, in this generalized statement a great deal that is interesting and important about extension and motion does not appear; and, so far as we know, these peculiar facts can be learned only by observation.

Now in order to make clear to ourselves what real extension is it will be necessary for us to perform in imagination some simple experiments. I say, 'in imagination' only, because most of the experiments in question have been actually performed by us many times, and the rest are familiar to us by report. A complete account of what real extension is would, indeed, necessitate the development of a complete system of elementary geometrical concepts; and I doubt very much whether I should be competent to do this. Let us therefore limit ourselves to a much simpler task, and ask ourselves what *linear* extension is; or, to be short and definite, what an *inch* is. If we can determine this, it will probably afford us a sufficiently instructive example of a primary quality.

One more preliminary remark must be made. When one is dealing with the pure sciences of geometry and mechanics, their relation of logical priority is very evident. Geometry can be thoroughly understood without the least reference to mechanics; but mechanics constantly presupposes geometry. In the experimental sciences this is not true. Geometrical and mechanical principles constantly presuppose each other. They are mutually complementary factors into which the complexity of experience has been analyzed.

That geometry is primarily concerned with solid bodies has often been observed. The gas expands to fill its container, and the liquid, flat on top, takes otherwise the shape of the vessel in which it lies. So that the application of geometry to gases and liquids is throughout dependent upon its treatment of solids. More precisely we may say that geometry has to do with *the conditions under which a solid which touches one of two given solids may also touch another.*

I shall not attempt to enumerate all the various experiences by which the distinction between solid and fluid bodies is impressed upon us. The solids resist the grasp of our hands, the bite of our teeth, as the others do not. We may add the fact (emphasized by Poincaré) that solids are frequently observed to move in such a fashion that the movement can be compensated by a movement of our own; that is to say, after we have moved, the solids present the same visible and tangible appearance as before. This is only exceptionally the case with fluids, and some solid almost always moves with the fluid when this happens. Again, I shall not attempt to enumerate the various ways in which we perceive that two solids are in contact with each other. Suffice it to note that when one of the two solids is the human body itself, a special sensation often informs us of the contact; and when we have hold of one solid and it touches another, an intensification of this sensation commonly results. Furthermore, when one solid touches another, we can frequently move one in such a fashion that the other moves simultaneously; and we can scarcely ever do this when the bodies are not in contact, unless indeed, some other solid is in contact with both.

It should be remarked in passing, that *solidity and contact are* *thought of as a real property and a real relation of certain things, and are not to be identified with any particular perception of ours.* (Either of them would have served, almost or quite as well as linear extension, as an example for the purpose of our study.) There are a considerable variety of perceptions which may convince us that an object is solid or that two objects are in contact; and not all these tests are always possible. But the tests that are at any time possible show a general agreement.

Let us now consider the following definition: Two solids A and B are said to be 'farther apart than' two solids C and D, when there exists a solid X which can be brought into simultaneous contact with (or made to 'connect') C and D but cannot be made to connect A and B. (It is understood that X can be freely moved, and that no other solid impedes its contact with the solids whose distance is being compared.) It can be proved by repeated trial that when this definition is satisfied, it is impossible to find any other solid Y such that it can connect A and B but cannot connect C and D. In other words, if A and B are farther apart than C and D, C and D are not farther apart than A and B. If every solid that can connect A and B can also connect C and D, and *vice versa*, then the two pairs of solids are said to be 'equidistant.' A class of equidistant pairs of objects, real or imaginary (or the property of being a member of such a class), is called a 'distance.'

Like other experimental definitions, these are necessarily modified in the sequel, by being extended to cases where experiment is impossible. That is to say, there are observed uniformities which enable us to predict, in advance of a direct trial, that one pair of objects are farther apart than another; and accordingly we extend the notion of 'farther apart than' to cases where a direct test not only has not yet been made but is impossible, and where, therefore, the original definition does not apply. Thus we say that the earth and Sirius are farther apart than the earth and the sun, though there is no solid that can be made to connect either pair. The statement means that any solid which *could* connect the earth and Sirius *would* also (according to the uniformities above referred to) be able to connect the earth and the sun; etc. Actual measurement is thus, in the development of the science, only the starting-point for indirect measurement.

Similarly, if there exist two solids A and B which the solid M can connect, but which the solid N cannot connect, M is said to be 'longer than' N. If each can connect every pair of solids that the other can connect, they are said to be 'equally long.' A class of equally long solids, real or imaginary (or the property of being a member of such a class), is called a 'length.'

It is obvious that to each length corresponds a certain distance, in such fashion that solids of the given length can connect pairs of solids at the given distance from each other. To this general statement, however, there is one exception. There are no solids whose lengths correspond to the distance between solids which are in contact with each other. To supply this lack we imagine an ideal class of solids called 'points' *whose length corresponds to the distance between solids that are in contact with each other;* and we assume that whenever any two solids are in contact with each other, at least one point is in contact with both. The length of a point is also called the 'zero-length.'[1]

It should not be supposed that this account of the length of solids purports to contain the only method by which solids can be compared as to their length. Such methods are indefinitely numerous. We are able to compare lengths directly by the eye and by touch and by the movement of the hand, as well as by the time consumed in various movements. But the method which I have described is, in principle, that which we regard as decisive (for the method of superposition is merely a simple case of it), and accordingly the definition of length has been based upon it.

Now, with this definition before us, certain familiar, but in our day frequently questioned truths become evident.

1. A definite length is not a property which belongs to one thing considered apart from other things. A single solid would

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[1] The experimental definition of the *sum of two lengths* is not without interest. The length of A is the sum of the lengths of B and C, if A can connect any pair of solids which B and C, *when in contact with each other*, can connect, but no others. That the ordinary laws of addition are satisfied by this definition may be proved inductively.

have no length. Lengths and distances are relative to each other. To say that all the lengths in the world increase or decrease together, and all the distances increase or decrease accordingly, is to say that no change at all occurs.

2. *Length is not a quality of any sensation or sensation-complex, visual, cutaneous, or kinæsthetic.* The lengths and distances of things are, however, conditions under which sensations take place. In other words, *the things which have the property of length are not sensations or sensation-complexes, but conditions of the possibility of sensations.*

3. It will be observed that I have not been able to define length except in terms of solidity and contact (and, we may add, of simultaneity). These other terms were not defined, but certain experiences were mentioned by which solidity and contact can generally or frequently be detected. This sort of description could of course be given for length itself, and in particular for the zero-length or point; and then solidity and contact might have been defined in terms of length. More generally we may say that the elementary physical properties and relations form a system of concepts which can be used almost indifferently to define each other, but which are evidently *incapable of definition in sensational terms.*

4. The definitions of distance and length are based upon a universal negative proposition, which is directly established only by an induction by simple enumeration. We said that if X can connect C and D but not A and B, no Y exists that can connect A and B but not C and D. How else can this be proved, except by repeated failures to prove the contrary? And it must be even noted that there are many experiences which, if uncritically accepted, do prove the contrary. But when such an experience occurs we hold fast to the general rule, and explain the exception by saying that one of the distances or one of the lengths changed —even though no otherwise apparent change has occurred. It is reflections of this sort that led Poincaré to hold that geometry cannot, like physics, be based upon experiment. But we should rather say that if things were everywhere moving and shrinking and expanding there would be no geometry. We must not forget

that, as far as ordinary observation can detect, most things are generally at rest. Geometry, indeed, is primarily concerned with things at rest—except as we, for the purpose of measurement, voluntarily move them.

5. This conception of length (including that extraordinary artificiality, the point), and, let us now add, a set of other conceptions based upon similarly imperfect induction, *works*. Which is as much as to say, first, that the deductions based upon them are found to lead to no contradiction, and, secondly, that the indirect measurements which they permit are, when tested by direct measurements, invariably confirmed. Not always exactly confirmed, to be sure, for that is not the way with experiments; but the discrepancies are not larger than might be expected from the discrepancies that arise in direct measurement itself; and there is no apparent tendency to error in any particular direction.

Let us now pass on to the further question (irrelevant to pure geometry but of great importance to applied geometry): What is an inch? for which may be substituted, What is a yard, what is a meter, or, in general, how is any particular length standardized? We know, of course, that all our standards of length are conventional, differing in this respect from the standard of angular measurement, for instance. Thus the proposition, that all right angles are equal, is significant, and is in fact proved in the geometry-books. But the proposition, that all inches are equal, is not significant in any sense that would permit of a general demonstration. It can only mean that all things assumed to be an inch long are equally long—which, indeed, is very doubtful. Again, if all right angles in the world were destroyed we should none the less be able to construct new ones. But if all the inch-measures, and all things whose lengths had known ratios to the inch were destroyed, we could never recover the inch. It is absolutely arbitrary.

How, then, we repeat, is the standard fixed? The answer is simple, but, I think, not uninstructive. We might be tempted to say that the standard is some one solid body, preserved for the sake of comparison, or a number of such bodies, whose lengths have been found by careful trial to show no distinguishable inequality. But this would not be accurate. For, to take the latter alternative first, we well know that things which at one time seem to be equal in length, at other times show obvious differences. And if, on the other hand, we propose to take a certain object as standard, we have to face the objection that its length may change—and what sense can be made of that? The problem may be put in another way. We say that the standard inch is the length of a certain piece of platinum, but we add that it must be at a certain temperature. What is our motive for this addition? We say that heat expands and cold contracts. But if this piece of platinum is the standard, how can *it* expand? Other things might contract when it grows warmer, but that is all. And yet that is ridiculous; for how could a change in the temperature of the platinum alter the length of anything else? *Just as in our measurement things in general are assumed to be at rest, so also things in general are assumed to remain unchanged in length.* Thus the given piece of platinum at a certain temperature is only the proximate standard. *The ultimate standard is the lengths of things in general.* This can be shown in another way. Suppose it should be discovered that platinum when left lying in a certain direction slowly takes on a crystalline structure like that of magnetized iron, and that its length in consequence appreciably increases; and suppose this had happened to our standard inch. There is nothing ridiculous in such a supposition. Our ultimate standard, the lengths of things in general, would simply require a further specification in the proximate standard: we should specify, say, that the platinum shall not be in the crystalline form, or choose some other metal instead.

But what sort of a standard is this—the lengths of things in general? It is incapable of exact application. The use of it is simply one more case of incomplete induction; and yet all natural science is based upon it.

Now it seems to me that a very similar account is to be given of the other fundamental modes of measurement employed in natural science. Suppose, for example, we ask what is meant by the duration of a physical process; or, again, more specifically, what a second is. In this case we have in the perception of motor and auditory rhythm a *prima facie* evidence of equality—just as in the visual perception of symmetry we have a *prima facie* evidence of equality in length. There is also, as we know, a perception of longer intervals, the psychological mechanism of which is little understood. But no more in the case of duration than in that of length do we limit the field of the relation to cases where such a perception is possible; nor, where the perception is possible, do we accept it as an ultimate criterion.

The measurement of time is, from the nature of the case, very different from that of space. The method of double contact is impossible; but, on the other hand, the one-dimensionality of time makes the procedure in some respects more simple. When the process *A* goes on all the time that *B* goes on, but the converse is not true, *A* takes longer than *B*. The problem is to compare processes where this relation does not hold. There appear to be two principal methods of advance. The one is based upon the observed rhythmical character of oscillations and vibrations. Durations are measured, for example, in terms of the oscillations of a pendulum of a given length. The other method is based upon the assumption, that when the same process is repeated under like circumstances, its duration is the same. Hence the use of the water-clock and the hour-glass and King' Alfred's lantern—yes, and the siderial revolution. In either case a specific duration is a class of processes that are equally long (or the property of belonging to such a class).

The comments which I would offer here are all similar to those offered in the case of linear measurement, with the notable exception of the fact that we perceive simultaneity between sensations and physical processes, and hence are capable of measuring the duration of the former by the latter. But, aside from this, we have again to note that the specific duration of a process is not a property which belongs to it independently of other processes, but a property which denotes its place in a system of relations. We have also to note that, defective as the observations are upon which the system of measurement is based, it works. Again, the duration is defined in terms (such as simultaneity) which cannot themselves be defined in psychical terms. For though we can, with a fair degree of accuracy observe simultaneity (as we can observe contact), we do not hesitate to assume that the perception is entirely unessential to the fact of simultaneity. We often affirm the fact where the perception is absent, and occasionally deny the fact when the perception is present. Finally, the standards of duration are conventional in exactly the same sense as the standards of length. We have a certain absolute temporal memory, as we have a certain absolute distance-memory; but both are inaccurate, and neither is as a matter of fact trusted with the maintenance of the standard.

That analogous remarks are to be made with respect to the physical measures of mass, force, and work will not be denied. We have a perception of force in the sense of strain. We have a perception of mass by means of the combined senses of strain and movement (tendinous and articular), as, for example, in rolling a billiard-ball. We have a perception of work in the sense of muscular fatigue. *But none of these sensations is for a moment identified with the perceived fact.* And what these facts are can only be defined in terms which are similarly incapable of definition in psychical terms. They must be defined, so to speak, in terms of each other. Like length, and unlike duration, they are not properties of any sensation or sensation-complex, though there exists the well-known correlation between the force of certain stimuli and the intensity of the resultant sensations. Their specific measures are wholly conventional, as are the measures of light and duration. Finally, the measures of length, duration, time, and force are united by the constant relation expressed by the formula, $f \cdot t = m \cdot a$, which is popularly known in the particular form of the 'law of falling bodies.' And with this formula to hold it together, the whole scheme *works*.

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